

ECONOMETRICS COMPREHENSIVE EXAM

JANUARY 16, 2007

Answer any five problems. Show your works. Good luck!!

Section A: Yoon

1. The variance of the maximum likelihood estimator of the $k \times 1$ parameter vector θ can be estimated in three different ways. Describe them giving their formulas.

2. The variable Y is censored in that

$$Y = a, \text{ if } Y^* \leq a,$$

$$Y = Y^*, \text{ if } Y^* > a.$$

Assuming that Y^* is normally distributed with mean μ and variance σ^2 , derive the expected value $E(Y)$ and variance $V(Y)$ of Y . You can use the results that $E(Y^*|Y^* > a) = \mu + \sigma \lambda$ and $V(Y^*|Y^* > a) = \sigma^2(1 - \delta)$.

You should provide the definitions of δ and λ

3. Consider the slope and intercept estimators b and a of the fixed effect model. T is the length of the panel data (number of periods) and n the number of individuals.

i) Examine consistency of b when $n \rightarrow \infty$. Justify your answer.

ii) Examine consistency of b with $T \rightarrow \infty$. Justify your answer.

Section B: Kumbhakar

4. Consider the regression of Y on K variables (with a constant), X , viz., $Y = X\beta + u$.

Consider an alternative set of regressors $Z = XP$ where P is a nonsingular design matrix of order K .

i) Prove that the residual vector in the regressions of Y on X and Y on Z are identical.

ii) Define a transformation matrix that expresses Y and X variables in mean deviation form and label them as y and x . Define another transformation matrix that will convert the y and x variables in standardized form (i.e., each of the variables will have zero mean and unit variance).

iii) Show that the effect of such transformation has no effect on R^2 , standard errors of β and their t values.

iv) Show that the the residual sum of squares, $e'e$ in the OLS regression Y on X and the regression y on x are the same.

5. In the generalized regression model $y = X\beta + u$, suppose that Ω is known

where $E(uu') = \sigma^2\Omega$. Let b and $\hat{\beta}$ are the OLS and GLS estimators of β .

a) What is the covariance matrix of the OLS and GLS estimator of β ?

b) What is the problem in using the OLS standard errors in hypothesis testing? How do you overcome this problem? Explain.

c) Can you think of a problem in real world when Ω is either known or can be consistently estimated?

d) What is the form of Ω if it is a time series model with AR(1) error (i.e., u follows AR(1) process)? If Ω is not known, can it be estimated? Explain.

6. Suppose you want to estimate the model

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t}^* + u_t, \quad (1)$$

where X_{2t}^* is not observed. (Both X_1 and X_2 are scalars). The measurement equation is

$$X_{2t} = X_{2t}^* + v_t, \quad (2)$$

where $E(u_t) = E(v_t) = 0$. $E(u_t^2) = \sigma_u^2$, $E(v_t^2) = \sigma_v^2$. v_t is independent of X_{2t}^* , X_{1t} and u_t .

Similarly u_t is independent of X_{2t}^* and X_{1t} .

- (a) Suggest an estimation procedure that will give you consistent estimators of β_0 , β_1 , and β_2 . Briefly explain.
- (b) Calculate the asymptotic bias in the OLS estimator of β_1 when X_{2t}^* is dropped from the model in (1).
- (c) Calculate the asymptotic bias in the OLS estimator of β_1 when X_{2t} is used as a proxy for X_{2t}^* in (1).
- (d) Comment on the results from (b) and (c) above.