ECONOMETRICS COMPREHENSIVE EXAM
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YOU MUST ANSWER QUESTION 1.

1. Consider the following model

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i, \quad i = 1, \ldots, N,$$

where (y_i, x_{i2}, x_{i3}) are observed and have finite moments, and ε_i is an unobserved error term. Suppose this model is estimated by ordinary least squares. Denote the OLS estimator by b.

a. What are the *essential* conditions required for unbiasedness of b? What are the *essential* conditions required for consistency of b? Explain the difference between unbiasedness and consistency.

b. Show how the conditions for consistency can be written as moment conditions. Explain how a method of moments estimator can be derived from these moment conditions. Is the resulting estimator any different from the OLS one?

Now suppose that $\text{cov}\{\varepsilon_i, x_{i3}\} \neq 0$.

c. Give an example of cases where one can expect a nonzero correlation between a regressor, x_{i3}, and the error ε_i. In this case, is it possible to still make appropriate inferences based on the OLS estimator, while adjusting the standard errors appropriately?

d. Explain how an instrumental variable, z_i, say, leads to a new moment condition and, consequently, an alternative estimator for β. Why does this alternative estimator lead to a smaller R^2 than the OLS one? What does this say of the R^2 as a measure for the adequacy of the model?

e. Why can we not choose $z_i = x_{i2}$ as an instrument for x_{i3}, even if $E\{x_{i2}\,\varepsilon_i\} = 0$? Would it be possible to use x_{i2}^2 as an instrument for x_{i3}?

ANSWER EITHER QUESTION 2 or 3

2. Let y_i denote the number of times individual i buys tobacco in a given month. Suppose a random sample of N individual is available, for which we observe values 0, 1, 2, 3,.... Let x_i be an observed characteristic of these individuals (e.g., gender). If we assume that, for given x_i, y_i has a Poisson distribution with parameter $\lambda_i = \exp\{\beta_1 + \beta_2 x_i\}$, the probability mass function of y_i conditional upon x_i is given by

$$P\{y_i = y | x_i\} = \frac{e^{-\lambda_i} \lambda_i^y}{y!}.$$

a. Write down the log-likelihood function (LLF) for this so-called Poisson regression model.

b. Derive the score contributions (first derivatives of the LLF with respect to the parameters). Using that the Poisson distribution implies that $E\{y_i | x_i\} = \lambda_i$, show that the score contributions have expectation zero.

3. Consider the partitioned regression model $y = X_1\beta_1 + X_2\beta_2 + u$.

a. Prove that the subvector $\hat{\beta}_1$ of the ordinary least-squares regression estimate $\hat{\beta} = (X'X)^{-1}X'y$, where $X = [X_1 \; X_2]$ is the set of coefficients obtained when the residuals from a regression of y on X_2 alone are regressed on the set of residuals obtained when each column of X_1 is regressed on X_2.

b. Show that the ordinary least-squares estimator of β in the model $y = \alpha\iota + X\beta + u$ is equal to the ordinary least-squares estimator of β in $y - \bar{y}\iota = [X - \iota\bar{x}]\beta + \varepsilon$, where α is the intercept, $\iota = [1,...,1]'$, $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$, and $\bar{x} = [\bar{x}_1,...,\bar{x}_k]$ with $\bar{x}_j = n^{-1}\sum_{i=1}^{n} x_{ji}; j = 1,...,k$.

c. What is the likely effect upon the ordinary least-squares estimate of β_1 in the model $y = X_1\beta_1 + X_2\beta_2 + u$ of omitting the variables in X_2?

ANSWER EITHER QUESTION 4 or 5.

4. Let $Y = X\beta + \varepsilon$, where Y is a nx1 dependent variable vector, X is a nxk matrix of independent variables, β is a vector of coefficients, and ε is a nx1 disturbance vector. Assume that errors terms are not homoscedastistic.

i) Derive V(b) where b is the least squares estimator of β.

ii) Write the estimator of V(b) suggested by H. White.

iii) Describe the White's test of heteroscedasticity, defining all your notations.

5. Let $\ln L(x;\theta)$ denote the log likelihood function where x is a nx1 *iid* random vector and θ is a 1x1 parameter.

i) Under regularity conditions, prove $\text{Var}\ \partial \ln L(x;\theta)/\partial\theta = -E\ \partial^2 \ln L(x;\theta)/\partial\theta^2$.

ii) Show three ways of estimating $\text{Var}\ \partial \ln L(x;\theta)/\partial\theta$.

YOU MUST ANSWER QUESTION 6.

6. To evaluate the benefit from a social program, the following model is employed:

$y_{1i} = X_i\beta_1 + u_{1i}$ (for program participants)
$y_{2i} = X_i\beta_2 + u_{2i}$ (for nonparticipants)
$I_i = Z_i\gamma + \varepsilon_i$ (participation decision function)

The observed y_i is defined as
$y_i = y_{1i}$ iff $I_i > 0$
$y_i = y_{2i}$ iff $I_i \leq 0$

Assume $(u_{1i}, u_{2i}, \varepsilon_i)$ is normally distributed with

$$Cov(u_{1i}, u_{2i}, \varepsilon_i) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{1\varepsilon} \\ \sigma_{12} & \sigma_{22} & \sigma_{2\varepsilon} \\ \sigma_{1\varepsilon} & \sigma_{2\varepsilon} & 1 \end{bmatrix}$$

i) Find $E(y_{1i} \mid I_i > 0)$ and $E(y_{2i} \mid I_i > 0)$

ii) Explain the statement "under self-selection those individuals who have a comparative advantage with the program will be joining the program and thus will benefit from it than would a randomly selected individual with the same characteristics."

iii) Prove the statement in ii).