#36 P.S.

ECONOMETRICS
COMPREHENSIVE EXAM
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ANSWER **ALL** QUESTIONS

1. Suppose that we have estimated the parameters of the multiple regression model:

$$Y_t = \beta_1 + \beta_2 X_{t2} + \beta_3 X_{t3} + u_t$$

by *Ordinary Least Squares* (OLS) method. Denote the estimated residuals by (e_t, $t = 1,\ldots,T$) and the predicted values by $(\hat{Y}_t\, t = 1,\ldots,T)$.

(a) What is the R^2 of the regression of e on a constant, X_2 and X_3?

(b) If we regress Y on a constant and $\hat{Y}$, what are the estimated intercept and slope coefficients? What is the relationship between the R^2 of this regression and the R^2 of the original regression?

(c) If we regress Y on a constant and e, what are the estimated intercept and slope coefficients? What is the relationship between the R^2 of this regression and the R^2 of the original regression?

(d) Suppose that we add a new explanatory variable X_4 to the original model and re-estimate the parameters by OLS. Show that the estimated coefficient of X_4 and its estimated standard error will be the same as in the OLS regression of e on a constant, X_2, X_3 and X_4.

2. Consider the simple regression with only a constant $y_i = \alpha + u_i$ for $i = 1,2,\ldots,n$; where the u_i's are independent with mean zero and var(u_i) = σ_1^2 for $i = 1, 2,\ldots, n_1$; and var (u_i) = σ_2^2 for $i = n_1 + 1,\ldots, n_1 + n_2$ with $n = n_1 + n_2$.

(a) Derive the OLS estimator of α along with its mean and variance.

(b) Derive the GLS estimator of α along with its mean and variance.

(c) Obtain the relative efficiency of OLS with respect to GLS. Compute their relative efficiency for various values of σ_2^2 / σ_1^2 = 0.4, 0.8, 1, 2.5; and n_1/n = 0.2, 0.4, 0.6, 0.8. Plot this relative efficiency.

(d) Assume that u_i is $N(0,\sigma_1^2)$ for i= 1,2, …, n_1 + 1, …, n_1 + n_2; with u_i's being independent. What is the maximum likelihood estimator of α, σ_1^2 and σ_2^2?

(e) Derive the LR test for testing H_0; $\sigma_1^2 = \sigma_2^2$ in part (d).

3. Consider the simple linear regression

$$y_i = \alpha + \beta X_i + u_i \qquad i = 1, 2, \ldots, n.$$

where α and β are scalars and u_i ~ IIN(0, σ^2). For H_0: $\beta = 0$,

(a) Derive the Likelihood Ratio (LR) statistic and show that it can be written as $n \ln [1/(1-r^2)]$ where r^2 is the square of the correlation coefficient between X and y.

(b) Derive the Wald (W) statistic for testing H_0: $\beta = 0$. Show that it can be written as $nr^2/(1-r^2)$. This is the square of the usual t-statistic on β with $\hat{\sigma}^2_{MLE} = \sum_{i=1}^{n} e_i^2 / n$ used instead of s^2 in estimating σ^2. $\hat{\beta}$ is the unrestricted MLE which is OLS in this case, and the e_i's are the usual least squares residuals.

(c) Derive the Lagrange Multiplier (LM) statistic for testing H_0: $\beta = 0$. Show that it can be written as nr^2. This is the square of the usual t-statistic on with $\hat{\sigma}^2_{RMLE} = \sum_{i=1}^{n} (y_i - \bar{y})^2 / n$ used instead of s^2 in estimating σ^2. The $\hat{\sigma}^2_{RMLE}$ is restricted MLE of σ^2 (i.e., imposing H_0 and maximizing the likelihood with respect to σ^2).

(d) Show that $LM/n = (W/n)/[1 + (W/n)]$, and $LR/n = \log[1 + (W/n)]$. Use the following inequality

x log (1 + x) $x/(1 + x)$, conclude that W LR LM. **Hint**: Use $x = W/n$.

4. Consider the following two-equation system:

$$y_1 + \beta_{12} y_2 = u_1 \qquad (1)$$
$$\beta_{21} y_1 + y_2 + \gamma_{21} x_1 = u_2 \qquad (2)$$

i) Check identifiability of equation (1) and equation (2). Justify your answer.

ii) Suppose you make an extra assumption that cov(u_1, u_2) = 0. How does this assumption affect the identifiability of equation (2)? Justify your answer.

5. Let $\ln L(X;\theta)$ denote the log likelihood function where X is a $n \times 1$ iid random vector and θ is a scalar. Assume the regularity conditions.

i) List the regularity conditions.

ii) Prove $E\, \partial \ln L(X;\theta)/\partial\theta = 0$.

iii) Consider an estimator $t(X)$ of θ where $E\{t(X)\} = \theta$. Prove that the sampling variance of $t(X)$ is at least as large as the Cramer-Rao lower bound.

6. A censored regression model or Tobit model is

$y_i{}^* = \beta' x_i + \varepsilon_i$, (3)

where $y_i = 0$ if $y_i{}^* < 0$, $y_i = y_i{}^*$ otherwise; $\varepsilon_i \sim N(0, \sigma^2)$.

i) Find $E(y_i)$.

ii) Derive the log likelihood function for model in (3).