#39

# ECONOMETRICS COMPREHENSIVE EXAM

August 23, 2005

Note: Please answer a total of **5 questions** with a **minimum of 1 from each section**.

**Section I: Henderson**

1. Consider the following stochastic processes:
   - (i) $y_t = \alpha + \beta t + \varepsilon_t$
   - (ii) $y_t = \alpha + \phi y_{t-1} + \varepsilon_t$
   - (iii) $y_t = \alpha + \phi y_{t-1} + \theta \varepsilon_{t-1} + \varepsilon_t$

   where $t$ is a deterministic trend and $\varepsilon_t \sim N(0, \sigma^2)$.
   - (a) Define the following concepts precisely:
     - (i) Stochastic Process
     - (ii) Stationarity
     - (iii) Invertibility
     - (iv) Optimal Forecast
   - (b) For each of the above processes, assess the conditions under which they are stationary and invertible. Calculate the conditional mean, the conditional variance, the unconditional mean and the unconditional variance. Calculate their autocorrelation functions up to lag 5.

2. Consider the second-order autoregressive process

   $y_t = c + \phi y_{t-2} + \varepsilon_t$

   where $\varepsilon_t \sim N(0, \sigma^2)$.
   - (a) Find:
     - (i) $E(y_t \mid \Omega_{t-2})$
     - (ii) $E(y_t \mid \Omega_{t-1})$
     - (iii) $E(y_{t+2} \mid \Omega_t)$
     - (iv) $\mathrm{Cov}(y_t, y_{t-1})$
     - (v) $\mathrm{Cov}(y_t, y_{t-2})$
     - (vi) Partial Autocorrelation Function
   - (b) Find $\partial y_t / \partial \varepsilon_t$, that is, trace the effects of a shock at time $t$ on the process $y_t$, $k$ periods ahead.
   - (c) Find the forecast $E(y_{t+s} \mid \Omega_t)$. What is the corresponding forecast error? Find the autocorrelation function of the forecast error.

3. Consider the following simultaneous equation model

$$y_{1t} + \beta y_{2t} = u_{1t} \qquad y_{1t} + \alpha y_{2t} = u_{2t}$$
$$u_{1t} = u_{1t-1} + \varepsilon_{1t} \qquad u_{2t} = \rho u_{2t-1} + \varepsilon_{2t}$$

where $|\rho| < 1$, and $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are uncorrelated white noise disturbances.

(a) What is the order of integration of $y_{1t}$? Why? Are $y_{1t}$ and $y_{2t}$ cointegrated? Why or why not. How does the idea of cointegration relate to the concept of equilibrium?

(b) How would you test for cointegration? State the null and the alternative hypothesis. Explain, at least, three test statistics, which are able to perform the testing.

(c) Suppose that $y_{1t}$ and $y_{2t}$ are cointegrated. What is the cointegrating vector and how can you estimate it? Derive the error correction representation of the model? Is it possible to estimate $\alpha$, $\beta$, and $\rho$?

**Section II: Yoon**

1. Consider the following earning model

$$\mathbf{y}_i = \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i$$

where $x_i$ is observed all the time but $y_i$ is observed only when $z_i^* > 0$, where $z_i^* = \gamma' w_i + u_i$. $u_i$ and $\varepsilon_i$ are assumed to have bivariate normal distribution with nonzero correlation between them.

(a) Find $E(y_i | z_i^* > 0)$ and $Var(y_i | z_i^* > 0)$.

(b) Show how you would estimate the parameter vector, $\boldsymbol{\beta}$.

(c) Show how to estimate the Variance-Covariance matrix of your $\boldsymbol{\beta}$ estimator in (b).

2. Let $\mathbf{m}_h'$ denote the sample moment and $\mu_h'$ the population moment around 0 of order h.

(a) Derive the variance of $\mathbf{m}_h'$ expressed in population moments.

(b) Is $\mathbf{m}_h'$ consistent? Justify your answer.

(c) Prove asymptotic normality of $\mathbf{m}_h'$.

**Section III: Kumbhakar**

1. Consider the standard *K* variable regression model $y = x\beta + u$.

(a) Explain the multicollinearity problem in terms of the above model. Is the OLS estimator of $\beta$ biased by the presence of multicollinearity?

(b) Give a realistic example of perfect multicollinearity. Is there a way to fix the problem in your example?

(c) Show that the variance of the OLS estimator of $\beta_k$ in a *K* variable regression model can be expressed as

$$V(b_k) = \frac{\sigma^2}{\sum x_k^2(1-R_k^2)} \equiv \frac{\sigma^2}{\sum x_k^2} VIF, k = 2,\dots,K$$

where VIF is called the variance inflation factor, $x_k$ is the *k*th regressor in deviation form, and $R_k^2$ is the $R^2$ of the auxiliary regression $X_k$ on all other *X* variables. What is the minimum and maximum value of VIF? How is VIF related to the severity of the multicollinearity problem?

2. Why is identification an issue in a simultaneous equation system? Consider the following system of demand and supply equations:

$$Demand: \quad Q = \alpha_1 P + \alpha_2 Z_1 + u_1$$
$$Supply: \quad Q = \beta_1 P + \beta_2 Z_2 + u_2$$

where Q and P are endogenous variables.

(a) Show (using both rank and order conditions) that the above system is identified.

(b) Derive the reduced form equations and show (give an intuitive explanation) that the parameters in the reduced form are consistently estimated.

(c) Suppose that you want to estimate the **demand function only**. Show that the OLS estimators of $\alpha_1$ and $\alpha_2$ from the demand function are inconsistent (give an intuitive explanation). Derive the instrumental variable estimators of $\alpha_1$ and $\alpha_2$ using $Z_1$ and $Z_2$ as instruments.